

Effects of shear and rotation on the spherical collapse model for clustering dark energy

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ABSTRACT

In the framework of the spherical collapse model we study the influence of shear and rotation terms for dark matter fluid in clustering dark energy models. We evaluate, for different equations of state, the effects of these terms on the linear overdensity threshold parameter, δ_c , and on the virial overdensity, Δ_V . The evaluation of their effects on δ_c allows us to infer the modifications occurring on the mass function. Due to ambiguities in the definition of the halo mass in the case of clustering dark energy, we consider two different situations: the first is the classical one where the mass is of the dark matter halo only, while the second one is given by the sum of the mass of dark matter and dark energy. As previously found, the spherical collapse model becomes mass dependant and the two additional terms oppose to the collapse of the perturbations, especially on galactic scales, with respect to the spherical non-rotating model, while on clusters scales the effects of shear and rotation become negligible. The values for δ_c and Δ_V are higher than the standard spherical model. Regarding the effects of the additional non-linear terms on the mass function, we evaluate the number density of halos. As expected, major differences appear at high masses and redshifts. In particular, quintessence (phantom) models predict more (less) objects with respect to the Λ CDM model and the mass correction due to the contribution of the dark energy component has negligible effects on the overall number of structures.

Key words: methods: analytical - cosmology: theory - dark energy

1 INTRODUCTION

One of the most complex puzzle in modern cosmology is the understanding of the nature of the accelerated expansion of the Universe. This astonishing fact is the result of observations of high-redshifts supernovae, that are less luminous of what was expected in a decelerated universe (Riess et al. 1998; Perlmutter et al. 1999; Tonry et al. 2003). Assuming General Relativity and interpreting the dimming of Type Ia supernovae as due to an accelerated expansion phase in the history of the Universe, we are forced to introduce a new component with negative pressure, and in particular, to cause accelerated expansion, its equation-of-state parameter must be $w < -1/3$. This fluid, usually dubbed *dark energy* (DE), is totally unknown in its nature and physical characteristics.

The latest observations of Supernovae type Ia (Riess et al. 1998; Perlmutter et al. 1999; Knop et al. 2003; Riess et al. 2004; Astier et al. 2006; Riess et al. 2007; Amanullah et al. 2010), together with the cosmic microwave

background (CMB) (Komatsu et al. 2011; Jarosik et al. 2011; Planck Collaboration et al. 2013a,b,c; Sievers et al. 2013), the integrated Sachs-Wolfe effect (ISW) (Giannantonio et al. 2008; Ho et al. 2008), the large scale structure (LSS) and baryonic acoustic oscillations (Tegmark et al. 2004a,b; Cole et al. 2005; Eisenstein et al. 2005; Percival et al. 2010; Reid et al. 2010; Blake et al. 2011), the globular clusters (Krauss & Chaboyer 2003; Dotter et al. 2011), high redshift galaxies (Alcaniz et al. 2003) and the galaxy clusters (Haiman et al. 2001; Allen et al. 2004, 2008; Wang et al. 2004; Basilakos et al. 2010) till works based on weak gravitational lensing (Hoekstra et al. 2006; Jarvis et al. 2006) and X-ray clusters (Vikhlinin et al. 2009) confirmed these early findings and they are all in agreement with a universe filled with 30% by cold dark matter and baryons (both fluids pressureless) and with the remaining 70% by the cosmological constant Λ (the so-called Λ CDM model). The cosmological constant is the most basic form of dark energy. Its equation of state is constant in time ($w = -1$), it appears in Einstein field equations as a geometrical term, it cannot cluster (being constant in space and time) and its importance is appreciable only at low redshift.

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Despite being in agreement virtually with all the observables, the standard cosmological model suffers of severe theoretical problems (the coincidence and the fine tuning problems) and therefore alternative models have been explored (but see also Astashenok & del Popolo 2012). The most studied ones are minimally coupled scalar fields (quintessence models). Since gravity is the main interaction acting on large scales, it is commonly believed that structures in the Universe formed via gravitational instability of primordial overdense perturbations that originated in the primeval inflationary phase (Starobinsky 1980; Guth 1981; Linde 1990) of the Universe from quantum, Gaussian distributed fluctuations (Del Popolo 2007; Komatsu 2010; Casaponsa et al. 2011; Curto et al. 2011; Komatsu et al. 2011; Hinshaw et al. 2013; Del Popolo 2014).

Differently from the cosmological constant, even if often neglected in literature, dynamical dark energy models possess fluctuations that can alter the evolution of structure formation, not only via slowing down the growth rate but also giving rise to DE overdensities and underdensities which can evolve into the non-linear regime.

To study structure formation in the highly non-linear regime, it is very useful to work within the framework of the spherical collapse model, introduced by Gunn & Gott (1972) and extended in many following works (Fillmore & Goldreich 1984; Bertschinger 1985; Hoffman & Shaham 1985; Ryden & Gunn 1987; Avila-Reese et al. 1998; Subramanian et al. 2000; Ascasibar et al. 2004; Mota & van de Bruck 2004; Williams et al. 2004; Abramo et al. 2007; Pace et al. 2010, 2014). According to the model, perturbations are considered as being spherically symmetric non-rotating objects that, being overdense, decouple from the background Hubble expansion, reach a point of maximum expansion (turn-around) and collapse (formally to a singularity). In reality this does not happen and the kinetic energy associated with the collapse is converted into random motions creating an equilibrium configuration (a virialized structure).

Despite its crude approximations, the model is very successful in reproducing results of N-body simulations when combined with mass the function formalism (Del Popolo 2007; Hiotelis & del Popolo 2013), either in usual minimally coupled dark energy models (Pace et al. 2010) or in non-minimally coupled dark energy models (Pace et al. 2014). Nevertheless it is important to extend the basic formalism to include additional terms and make it more realistic.

Consequences of relaxing the sphericity assumption were studied by Hoffman (1986, 1989) and Zaroubi & Hoffman (1993), while the introduction of radial motions and angular momentum was deeply studied by Ryden & Gunn (1987) and Gurevich & Zybin (1988a,b). We refer to Del Popolo et al. (2013b) for a more complete list of references and for details on the different models and how to link the angular momentum to the matter overdensity.

In this work we will extend previous works on the extended spherical collapse model in dark energy models (Del Popolo et al. 2013a,b,c) by taking into account perturbations of the DE fluid. Since there are no N-body simulations with clustering dark energy so far, such study is valuable in order to have an idea about how dark energy fluctuations impact structure formation in a more realistic scenario. By writing the differential equations describing the dynamics of dark matter and dark energy, we will show how to relate the additional terms (shear and angular momentum) to the overdensity field and we will solve them to derive the time evolution of the typical parameters of the spherical collapse model, in particular the linear overdensity threshold for collapse δ_c and the virial over-

density Δ_V and we will show how these quantities are modified by the introduction of non zero vorticity and shear terms. Afterwards we will show how the mass function and its phenomenological extension to include DE perturbations are affected.

The paper is organised as following. In Section 2 we discuss and summarise the dark energy models used in this work. In Section 3 we briefly derive the equations of the extended spherical collapse model whose solution will lead to the evaluation of δ_c and Δ_V (see Section 4.1). In Section 4 we show our results and in particular we devote Section 4.2 to the discussion of the effects of shear and rotation on the mass function. Finally, we conclude in Section 5.

2 THE MODELS

For this work we use dark energy models previously analysed in the framework of the spherical collapse model where the usual assumption of negligible dark energy fluctuations is relaxed.

Dark energy models, described by an equation of state $w = P/(\rho c^2)$, either constant or time dependant, satisfy the background continuity equation

$$\dot{\rho} + 3H(1+w)\rho = 0. \quad (1)$$

We consider eight different models and for the ones characterised by an evolving equation-of-state parameter we adopted the Chevallier & Polarski (2001) and Linder (2003) (CPL) linear parametrization

$$w(a) = w_0 + (1-a)w_a, \quad (2)$$

where w_0 and w_a are constants and a is the scale factor.

The reference model is the standard Λ CDM model where dark energy is represented by the cosmological constant with equation-of-state parameter $w = -1$, constant along the whole cosmic history. A consequence of this parametrization is that at early times this model behaves essentially as the EdS model (with $\Omega_m = 1$ and $\Omega_{de} = 0$) and the influence of the cosmological constant becomes appreciable only late in the cosmic history.

Due to the latest observational results by the *Planck* satellite¹ (Planck Collaboration et al. 2013a,b,d), we will assume a spatially-flat model.

Of the remaining seven models, two have a constant equation-of-state parameter $w > -1$ (the quintessence models DE1 and DE2) and they differ from each other solely for the exact value of w . Other two instead have $w < -1$ (the phantom models DE3 and DE4). The latter are justified by recent Supernovae Type Ia (SNIa) observations (Novosyadlyj et al. 2012; Planck Collaboration et al. 2013b; Rest et al. 2013; Scolnic et al. 2013; Shafer & Huterer 2014).

Finally we consider three additional models with a time varying equation of state. Once again we can distinguish them according to the general behaviour of the equation-of-state parameter. One model enters in the quintessence model category (DE5), the second one is a phantom model (DE6) and the last one (DE7) is characterised by the barrier crossing, i.e., the model considered shows a phantom regime at low redshifts ($w < -1$) and a quintessence regime at earlier times ($w > -1$).

As previously stated, quintessence models are described by a scalar field not interacting with matter and are fully described by a kinetic and a potential energy term. Since the nature of dark

¹ <http://sci.esa.int/planck/>

Table 1. Values of the parameters describing the equations of state considered in this work.

Model	w_0	w_a
Λ CDM	-1	0
DE1	-0.9	0
DE2	-0.8	0
DE3	-1.1	0
DE4	-1.2	0
DE5	-0.75	0.4
DE6	-1.1	-1.
DE7	-1.1	0.5

energy is unknown, the potential has an ad hoc functional form and its second derivative represents the mass of the scalar field. These models naturally have an evolving dark energy equation of state w . Scalar fields are therefore viable candidates for the dark energy component.

Phantom models instead have $w < -1$ and challenge the foundations of theoretical physics violating several energy conditions. Phantom models have a negative defined kinetic energy term and due to the super-negative equation of state, the energy budget of the Universe gets completely dominated by them in the future.

The solution of Equation 1 is (for a generic time-dependant equation-of-state parameter $w(a)$)

$$\rho(a) = \rho(a=1)e^{-3 \int_1^a [1+w(a')] d \ln a'}. \quad (3)$$

In the particular case of constant equation of state, Equation 3 reduces to

$$\rho(a) = \rho(a=1)a^{-3(1+w)}, \quad (4)$$

where it appears clearly that for the cosmological constant $\rho(a) = \rho(a=1)$, hence the name.

In this work we will use the following cosmological parameters (recall that the curvature is null): $\Omega_m = 0.32$ and $\Omega_{de} = 0.68$, while $h = 0.72$, in agreement with recent determinations by Planck (Planck Collaboration et al. 2013b) for flat Λ CDM models. The normalization of the power spectrum for the Λ CDM model is $\sigma_8 = 0.776$.

In table 1 we give the values of the parameters describing the equations of state of the models considered here both for a null time evolution (models DE n , with n from 1 to 4) and for a time evolution (models DE n , with n from 5 to 7) of the equation of state. We recall that for $w_a = 0$, the CPL parametrization reduces to a constant equation of state with $w = w_0$. We show the time evolution of the equation-of-state parameter $w(z)$ in Fig. 1.

For the models DE n , with $n = 5, 6, 7$, the variation of the equation of state as a function of the redshift is quite mild, with major variations for $z \lesssim 2$. At higher redshifts all the three models reach a constant value for the equation of state, being $w = -0.35, -2, -0.6$ for the models DE5, DE6, DE7, respectively. We also point out that the barrier crossing for the model DE7 takes place for $z \approx 0.25$, having a phantom (quintessence) behaviour for smaller (higher) redshifts. We checked that all the models do not have an appreciable amount of dark energy at early times, therefore they can not be considered as belonging to the class of early dark energy models.

Perturbations for dark energy are described by the effective sound speed c_{eff}^2 that relates density perturbations to pressure perturbations via the relation $\delta p = c_{\text{eff}}^2 \delta \rho c^2$. In the follow-

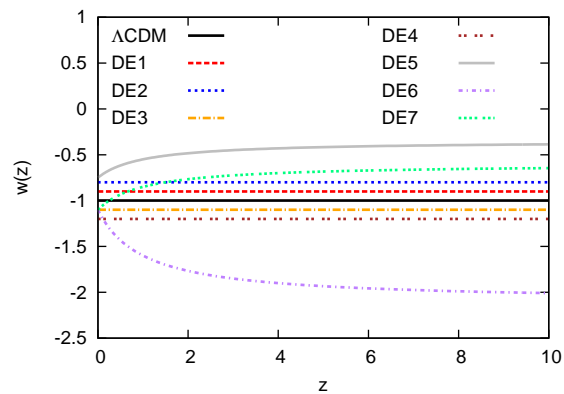


Figure 1. Time evolution of the equation-of-state parameter as a function of the redshift z for the dark energy models studied in this work. The black, red, blue, orange and brown lines represent the Λ CDM, the DE1, the DE2, the DE3 and the DE4 model, respectively. The grey line represents the DE5 model, the purple line the DE6 model and the green line the DE7 model.

ing we will consider two different values for the effective sound speed, usually assumed in literature: $c_{\text{eff}}^2 = 0$ (clustering DE) and $c_{\text{eff}}^2 = 1$ (smooth DE). Canonical scalar fields have $c_{\text{eff}}^2 = 1$, whereas models with vanishing c_{eff}^2 can be build from k-essence models (Armendariz-Picon et al. 2001; Chimento & Lazkoz 2005; Creminelli et al. 2009) or two scalar fields (Lim et al. 2010). We will show in Sect. 3 how this term enters into the equations for the (extended) spherical collapse model.

3 EXTENDED SPHERICAL COLLAPSE MODEL (ESCM)

In this section we review the basic formalism used to derive the equations for the spherical collapse model and how this can be extended to include shear and rotation terms.

The basic assumption in the framework of the spherical collapse model is that objects form under the gravitational collapse of spherical dark matter overdense perturbations. This is clearly a rather crude assumption because it is known that primordial seeds are not spherical, but they are triaxial and rotate (see e.g. Bardeen et al. 1986; Del Popolo et al. 2001; Del Popolo 2002; Shaw et al. 2006; Bett et al. 2007). Nevertheless the model accurately reproduces the results of N-body simulations.

The spherical and ellipsoidal collapse models were extensively investigated in literature (see, e.g. Bernardeau 1994; Bardeen et al. 1986; Ohta et al. 2003, 2004; Basilakos et al. 2009; Pace et al. 2010; Basilakos et al. 2010) assuming that dark energy perturbations are negligible, while other studies took into account also the effects of perturbations for the dark energy fluid (see Mota & van de Bruck 2004; Nunes & Mota 2006; Abramo et al. 2007, 2008, 2009,b; Creminelli et al. 2010; Basse et al. 2011; Batista & Pace 2013). More recently, the spherical collapse model was extended to investigate coupled (Pettorino & Baccigalupi 2008; Wintergerst & Pettorino 2010; Tarrant et al. 2012) and extended dark energy (scalar-tensor) models (Pettorino & Baccigalupi 2008; Pace et al. 2014).

While the general equations including the shear and rotation were explicitly written in the case of smooth dark energy (Pace et al. 2010) and for clustering dark energy (Abramo et al. 2007), the effects of these two nonlinear terms were investigated only recently in Del Popolo et al. (2013a,b) for smooth dark en-

ergy models and in Del Popolo et al. (2013c) for Chaplygin cosmologies.

Following Abramo et al. (2007) and Abramo et al. (2008), the full perturbed equations describing the evolution of the dark matter (δ_{DM}) and dark energy (δ_{DE}) perturbations are:

$$\delta'_{\text{DM}} + (1 + \delta_{\text{DM}}) \frac{\theta_{\text{DM}}}{aH} = 0, \quad (5)$$

$$\delta'_{\text{DE}} - \frac{3}{a} w \delta_{\text{DE}} + [1 + w + \delta_{\text{DE}}] \frac{\theta_{\text{DE}}}{aH} = 0. \quad (6)$$

In the previous equations, w represents the equation of state of dark energy at the background level and the prime is the derivative with respect to the scale factor. The two variables θ_{DM} and θ_{DE} are the divergence of the peculiar velocity for the dark matter and the dark energy component, respectively. Equation 6 is valid in the limit $c_{\text{eff}}^2 = 0$, the limit of clustering dark energy. For the case $c_{\text{eff}}^2 > 0$, dark energy perturbations are usually negligible on small scales, as shown for example in Batista & Pace (2013).

To determine the equation for the evolution of the divergence of the peculiar velocity we have to make some assumptions on the influence of the shear and rotation terms on the perturbations of the two fluids considered. If we assume that only dark matter experiences shear and rotation terms, then the two peculiar velocities are different ($\theta_{\text{DM}} \neq \theta_{\text{DE}}$) and we will have two different equations: one including shear and rotation for the dark matter and one for the dark energy component without the extra terms. If instead we assume that DE experiences the effects of the shear and the rotation terms in the same fashion of the DM component, then the two peculiar velocities will be the same and we need to solve a single differential equation.

Here we explicitly write the two different equations for the peculiar velocities and in the next sections we will study the consequences of this assumption. Having therefore two different Euler equations, the equations for the divergence of the peculiar velocities are:

$$\theta'_{\text{DM}} + \frac{2}{a} \theta_{\text{DM}} + \frac{\theta_{\text{DM}}^2}{3aH} + \frac{\sigma_{\text{DM}}^2 - \omega_{\text{DM}}^2}{aH} + \frac{3H}{2a} [\Omega_{\text{DM}} \delta_{\text{DM}} + \Omega_{\text{DE}} \delta_{\text{DE}}] = 0 \quad (7)$$

$$\theta'_{\text{DE}} + \frac{2}{a} \theta_{\text{DE}} + \frac{\theta_{\text{DE}}^2}{3aH} + \frac{3H}{2a} [\Omega_{\text{DM}} \delta_{\text{DM}} + \Omega_{\text{DE}} \delta_{\text{DE}}] = 0 \quad (8)$$

The shear tensor σ_{ij} and the vorticity tensor ω_{ij} are defined as

$$\sigma_{ij} = \frac{1}{2} \left(\frac{\partial u^j}{\partial x^i} + \frac{\partial u^i}{\partial x^j} \right) - \frac{1}{3} \delta_{ij} \theta, \quad (9)$$

$$\omega_{ij} = \frac{1}{2} \left(\frac{\partial u^j}{\partial x^i} - \frac{\partial u^i}{\partial x^j} \right). \quad (10)$$

The terms σ^2 and ω^2 represent the contractions of the tensors σ_{ij} and ω_{ij} , respectively.

It is convenient to consider a dimensionless divergence of the comoving peculiar velocity, defined as $\tilde{\theta} = \theta/H$. Therefore Eqs. 5-8 read now

$$\delta'_{\text{DM}} + (1 + \delta_{\text{DM}}) \frac{\tilde{\theta}_{\text{DM}}}{a} = 0, \quad (11)$$

$$\delta'_{\text{DE}} - \frac{3}{a} w \delta_{\text{DE}} + [1 + w + \delta_{\text{DE}}] \frac{\tilde{\theta}_{\text{DE}}}{a} = 0, \quad (12)$$

$$\tilde{\theta}'_{\text{DM}} + \left(\frac{2}{a} + \frac{H'}{H} \right) \tilde{\theta}_{\text{DM}} + \frac{\tilde{\theta}_{\text{DM}}^2}{3a} + \frac{\tilde{\sigma}_{\text{DM}}^2 - \tilde{\omega}_{\text{DM}}^2}{a} + \frac{3}{2a} [\Omega_{\text{DM}} \delta_{\text{DM}} + \Omega_{\text{DE}} \delta_{\text{DE}}] = 0, \quad (13)$$

$$\tilde{\theta}'_{\text{DE}} + \left(\frac{2}{a} + \frac{H'}{H} \right) \tilde{\theta}_{\text{DE}} + \frac{\tilde{\theta}_{\text{DE}}^2}{3a} + \frac{3}{2a} [\Omega_{\text{DM}} \delta_{\text{DM}} + \Omega_{\text{DE}} \delta_{\text{DE}}] = 0. \quad (14)$$

We remind the reader that this set of equations is valid when dark energy is not affected by shear and rotation, otherwise Eqs. 13 and 14 will be identical and $\theta_{\text{DM}} = \theta_{\text{DE}}$.

To solve the system of equations 11, 12, 13 and 14, it is necessary to determine the initial conditions. At early times, the aforementioned system of equations can be linearised and it reads

$$\delta'_{\text{DM}} = -\frac{\tilde{\theta}_{\text{DM}}}{a}, \quad (15)$$

$$\delta'_{\text{DE}} - \frac{3}{a} w \delta_{\text{DE}} = -(1 + w) \frac{\tilde{\theta}_{\text{DE}}}{a}, \quad (16)$$

$$\tilde{\theta}'_{\text{DM}} + \left(\frac{2}{a} + \frac{H'}{H} \right) \tilde{\theta}_{\text{DM}} = -\frac{3}{2a} [\Omega_{\text{DM}} \delta_{\text{DM}} + \Omega_{\text{DE}} \delta_{\text{DE}}] \quad (17)$$

$$\tilde{\theta}'_{\text{DE}} + \left(\frac{2}{a} + \frac{H'}{H} \right) \tilde{\theta}_{\text{DE}} = -\frac{3}{2a} [\Omega_{\text{DM}} \delta_{\text{DM}} + \Omega_{\text{DE}} \delta_{\text{DE}}] \quad (18)$$

Hence at the linear level, the peculiar velocity perturbations are identical for both fluids.

The initial value for the dark matter overdensity can be found as outlined in Pace et al. (2010, 2012, 2014) and Batista & Pace (2013). Here we just recall the general procedure. Since at collapse time a_c the collapsing sphere reduces to a point, its density is formally infinite. Therefore the initial overdensity $\delta_{\text{DM},i}$ is given by the value such that $\delta_{\text{DM}} \rightarrow +\infty$ for $a \rightarrow a_c$. Knowing $\delta_{\text{DM},i}$ and assuming that at early times it behaves as a power law, $\delta_{\text{DM}} = Aa^n$, it is easy to evaluate the initial amplitude for the dark energy and the peculiar velocity perturbations:

$$\delta_{\text{DE},i} = \frac{n}{(n - 3w)} (1 + w) \delta_{\text{DM},i}, \quad (19)$$

$$\tilde{\theta}_{\text{DM},i} = -n \delta_{\text{DM},i}. \quad (20)$$

$$\tilde{\theta}_{\text{DE},i} = \tilde{\theta}_{\text{DM},i}. \quad (21)$$

For an EdS model, $n = 1$, but in general deviations for DE models are very small, even for early dark energy models (Ferreira & Joyce 1998; Batista & Pace 2013).

To evaluate the functional form of the term $\sigma_{\text{DM}}^2 - \omega_{\text{DM}}^2$ we refer to the works by Del Popolo et al. (2013a,b) and we define the quantity α as the ratio between the rotational and the gravitational term

$$\alpha = \frac{L^2}{M^3 R G}, \quad (22)$$

where M and R are the mass and the radius of the spherical overdensity respectively and L its angular momentum. Values for α range from 0.05 for galactic masses ($M \approx 10^{11} M_{\odot}/h$) to 3×10^{-6} for cluster scales ($M \approx 10^{15} M_{\odot}/h$).

As explained in Del Popolo et al. (2013b) the basic assump-

tion here made is that the collapse preserves the value of the ratio of the acceleration between the shear rotation term and the gravitational field. This is a reasonable assumption as explained in Del Popolo et al. (2013b). As shown in Del Popolo et al. (2013c), based on the above outlined argument for the definition of the rotation term, the additional term in the equations for the spherical collapse model (see Equation 13) is

$$\tilde{\sigma}_{\text{DM}}^2 - \tilde{\omega}_{\text{DM}}^2 = -\frac{3}{2}\alpha\Omega_{\text{DM}}\delta_{\text{DM}}. \quad (23)$$

According to this Ansatz, Equation 13 now reads

$$\tilde{\theta}'_{\text{DM}} + \left(\frac{2}{a} + \frac{H'}{H}\right)\tilde{\theta}_{\text{DM}} + \frac{\tilde{\theta}_{\text{DM}}^2}{3a} + \frac{3}{2a}[(1-\alpha)\Omega_{\text{DM}}\delta_{\text{DM}} + \Omega_{\text{DE}}\delta_{\text{DE}}] = 0. \quad (24)$$

If instead also dark energy is affected by shear and rotation, in the same way as dark matter, both velocity fields are determined by the equation:

$$\tilde{\theta}' + \left(\frac{2}{a} + \frac{H'}{H}\right)\tilde{\theta} + \frac{\tilde{\theta}^2}{3a} + \frac{3}{2a}(1-\alpha)[\Omega_{\text{DM}}\delta_{\text{DM}} + \Omega_{\text{DE}}\delta_{\text{DE}}] = 0. \quad (25)$$

4 RESULTS

In this section we present results for the linear and nonlinear evolution of perturbations. We first start with quantities derived in the framework of the spherical collapse model and we continue with a discussion of how rotation and shear affect the mass function in clustering dark energy cosmologies.

As shown in Batista & Pace (2013), the main difficulty is to study the evolution of dark energy perturbations in the non-linear regime (see also Mota & van de Bruck 2004; Nunes & Mota 2006; Abramo et al. 2007; Creminelli et al. 2010; Basse et al. 2011).

Batista & Pace (2013) clearly demonstrated that dark energy fluctuations are very sensitive to the value of c_{eff}^2 : when $c_{\text{eff}}^2 = 1$, on small scales, where non-linear evolution is important, dark energy fluctuations are negligible with respect to the dark matter fluctuations δ_{DM} therefore ignoring them when solving the system of equations describing the ESCM (Equations 11, 12, 14 and 24) does not introduce any significant error. Different is the situation when $c_{\text{eff}}^2 = 0$ since DE fluctuations can be comparable to the DM ones. In this case we cannot neglect them, otherwise the error introduced will be significant and invalidate our results and conclusions.

4.1 Parameters of the ESCM

The two main quantities that can be evaluated working within the framework of the ESCM are the linear overdensity parameter δ_c and the virial overdensity Δ_v . The linear overdensity parameter is a fundamental theoretical quantity entering, together with the linear growth factor, into analytical formulations of the mass function (see e.g. Press & Schechter 1974; Sheth et al. 2001; Sheth & Tormen 2002). The virial overdensity instead, is used to define the size of halos when considered spherical. Given a halo of mass M , it represents the mean density enclosed in the radius R and the mass and the radius are related to each other via the relation $M = 4/3\pi\bar{\rho}(z)\Delta_v(z)R^3$ where $\bar{\rho}(z) = \bar{\rho}_0(1+z)^3$ is the mean matter density in the Universe.

Once the initial conditions are found, we can evolve Equations 15-18 from the initial time $a_i \approx 10^{-5}$ to the collapse time a_c .

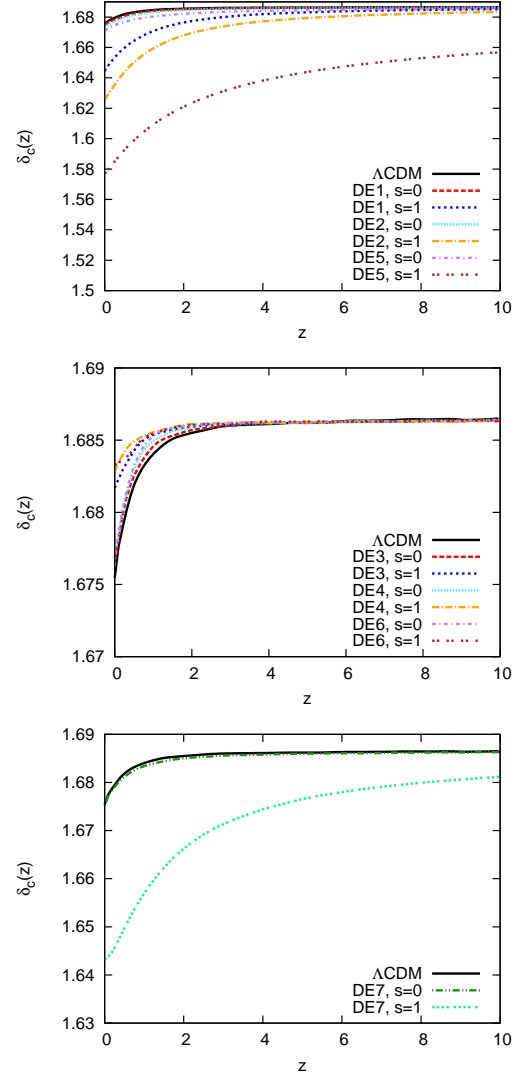


Figure 2. Linear overdensity parameter $\delta_c(z)$ for quintessence (upper panel), phantom (middle panel) and models with barrier crossing (bottom panel). In the upper panel, model DE1 with $c_{\text{eff}}^2 = 0$ ($c_{\text{eff}}^2 = 1$) is shown with a dashed red (blue short dashed) curve, model DE2 with $c_{\text{eff}}^2 = 0$ ($c_{\text{eff}}^2 = 1$) is shown with cyan dotted (yellow dot-dashed) curve, model DE5 with $c_{\text{eff}}^2 = 0$ ($c_{\text{eff}}^2 = 1$) with a violet dotted short-dashed (brown dot-dotted) curve. In the middle panel, model DE3 with $c_{\text{eff}}^2 = 0$ ($c_{\text{eff}}^2 = 1$) is shown with a red dashed (blue short-dashed) curve, model DE4 with $c_{\text{eff}}^2 = 0$ ($c_{\text{eff}}^2 = 1$) is shown with a cyan dotted (yellow dot-dashed) curve while model DE6 with $c_{\text{eff}}^2 = 0$ ($c_{\text{eff}}^2 = 1$) is shown with violet dotted short-dashed (brown dot-dotted) curve. In the bottom panel, model DE7 with $c_{\text{eff}}^2 = 0$ ($c_{\text{eff}}^2 = 1$) is shown with a dark green dot-dotted short-dashed (light green dot-dot-dotted) curve. In all the panels, a solid black line shows the Λ CDM model. For simplicity, we used the notation $s = c_{\text{eff}}^2$ in the labels.

This function therefore depends on both the linear and non-linear evolution of perturbations.

In Figure 2 we show the time evolution of the linear overdensity parameter δ_c for the usual case when shear and rotations are not included. We do so in order to better show how the additional terms modify this function. We show our results grouping the models as quintessence (top panel), phantom (middle panel) and barrier crossing models (bottom panel). We refer to the caption for line styles and colours of each model.

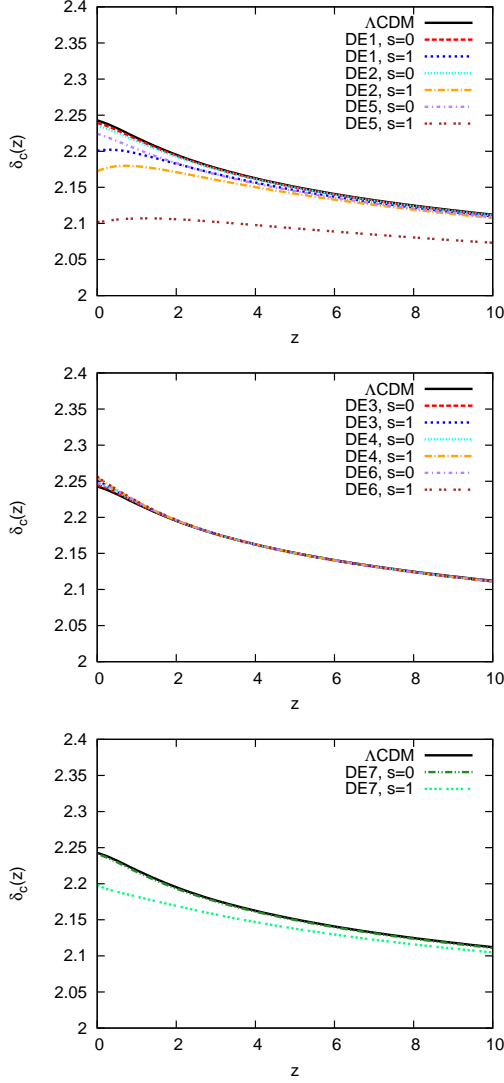


Figure 3. Linear overdensity parameter $\delta_c(z)$ for quintessence (upper panel), phantom (middle panel) and models with barrier crossing (bottom panel) including the shear and rotation terms in the equations for the evolution of the perturbations. Panels refer to galactic scale mass objects ($M \approx 10^{11} M_\odot/h$), corresponding to $\alpha = 0.05$. Line styles and colours are the same as in Fig. 2.

The first important point to highlight is that quintessence models ($w \geq -1$) always show a lower $\delta_c(z)$ with respect to the Λ CDM model, while the phantom models always have a higher value, due to the fastest accelerated expansion of the universe, that obstacles structure formation. We also notice that models with $c_{\text{eff}}^2 = 0$ are more similar to the Λ CDM model than for the case with $c_{\text{eff}}^2 = 1$, in agreement with what was found by Batista & Pace (2013) for early dark energy models, where we refer for a deeper explanation. For our purposes, it suffices to recall that this happens because DE perturbations contribute to the gravitational potential via the Poisson equation. Model DE7 has a very similar behaviour to the other classes of models, in particular to the quintessence models. The linear overdensity parameter is smaller than the one for the Λ CDM model. When $c_{\text{eff}}^2 = 0$, this model is almost identical to the Λ CDM model, while it differs substantially when $c_{\text{eff}}^2 = 1$. All the models with effective sound speed $c_{\text{eff}}^2 = 0$, converge very rapidly ($z \gtrsim 3$) to the Λ CDM model and hence to the EdS model, since DE be-

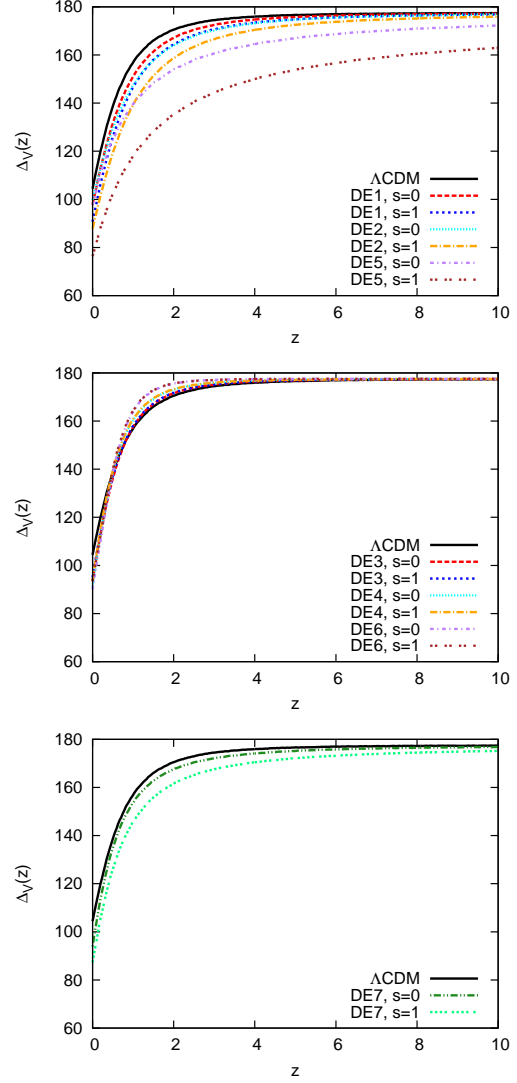


Figure 4. Virial overdensity for the difference DE models. In the upper (middle) panel we show results for the quintessence (phantom) models while in the bottom panel we present results for the model with barrier crossing. Line styles and colours are as in Figure 2.

comes negligible at such high redshifts. Different again is the situation for the $c_{\text{eff}}^2 = 1$ case, where the DE models recover the Λ CDM model at much higher redshifts (quintessence models), while phantom models reproduce the reference model very quickly. Models DE5 and DE7 with $c_{\text{eff}}^2 = 1$ instead do not recover the Λ CDM model even at high redshifts. As said before, this is largely due to the additional source for the gravitational potential and largely independent of the background equation of state w , as shown in Pace et al. (2010).

In Figure 3 we show results for δ_c when the shear and rotation terms are taken into account for the DM Euler equation. As already shown and discussed in Del Popolo et al. (2013a,b,c), the main effect appears at galactic scales ($M \approx 10^{11} M_\odot/h$). We verified that this is indeed the case also for clustering dark energy models, therefore we will limit ourselves to present our results for galactic scale objects. The differences between the spherical collapse model and the extended spherical collapse model become increasingly smaller with increasing mass, disappearing at cluster scales. Qualitatively, therefore, clustering and non-clustering dark energy models behave

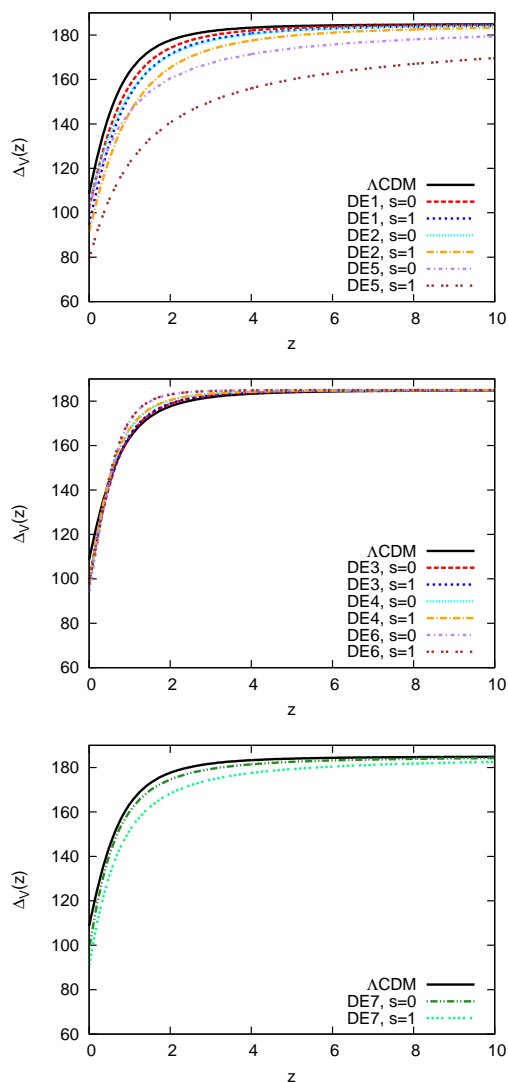


Figure 5. Virial overdensity for the difference DE models with shear and rotation terms included. In the upper (middle) panel we show results for the quintessence (phantom) models while in the bottom panel we present results for the model with barrier crossing. Panels refer to galactic scale mass objects ($M \approx 10^{11} M_{\odot}/h$), corresponding to $\alpha = 0.05$. Line styles and colours are as in Figure 2.

in the same way with respect to the mass dependence. We refer to the caption for line styles and colours of each model.

As expected, and in analogy with the extended spherical collapse model, when the influence of DE is only at the background level (Del Popolo et al. 2013b), the additional term opposes to the collapse, therefore the values for the linear overdensity parameter are higher than for the case in which these terms are neglected. Also in this case, quintessence models with $c_{\text{eff}}^2 = 1$ differ more from the Λ CDM model than for the case with $c_{\text{eff}}^2 = 0$. Phantom models are now very similar to the Λ CDM model, differently from before. Differences between the case with $c_{\text{eff}}^2 = 0$ and $c_{\text{eff}}^2 = 1$ are now negligible. Model DE7 behaves qualitatively as for the standard spherical collapse model. Also in this case all the models, except for the models DE5 and DE7 with $c_{\text{eff}}^2 = 1$, recover the Λ CDM model at high redshifts. As shown in Del Popolo et al. (2013a,b), in the ESCM major differences take place at low redshifts. We can therefore conclude that clustering DE models behave similarly to the

non-clustering DE models when shear and rotations are included in the analysis.

However DE and its perturbations can also affect the virialization process of dark matter. A reference work focusing on this issue is Maor & Lahav (2005). In this seminal work it was shown that a different result for the ratio between the virialization radius and the turn-around radius (the radius of maximum expansion) y changes according to the recipe used, in particular if the dark energy takes part or not into the virialization process. Whatever is the correct formulation for the virialization process in clustering DE models, our ignorance on the exact value of y will not qualitatively affect our discussion and conclusions, therefore for simplicity we will use $y = 1/2$, as in the Einstein-de Sitter model (see also the discussion in Batista & Pace 2013). Since clustering dark energy does not alter the temporal evolution of the dark matter energy density parameter, we can still write $\Delta_v = \zeta(x/y)^3$, where ζ represents the non-linear overdensity at turn-around, x is the scale factor normalised at the turn-around scale factor. Our results for the (non-)rotating case are presented in Figure (4) 5.

As before, we limit ourselves to the study of the effects of the shear and rotation terms at galactic scales, since this is the mass scale where the effect is stronger. As for the δ_c parameter, also in this case the DE models differ mostly from the reference model when the effective sound speed is of the order unity, while for $c_{\text{eff}}^2 = 0$ the models are closer to the Λ CDM model. We also notice that, since at high redshifts the amount of dark energy is negligible, DE models recover the Λ CDM model. The model differing more is, once again, the DE5 with $c_{\text{eff}}^2 = 1$ (see Figure 2). Quintessence (phantom) models have lower (higher) values of Δ_v with respect to the Λ CDM model. These results are qualitatively similar to what found in Pace et al. (2010). Model DE7 behaves like the quintessence models having slightly smaller values for the case $c_{\text{eff}}^2 = 0$.

We find qualitatively similar results in the ESCM (see Figure 5). With respect to the usual case, we observe, as expected, that the virial overdensity is higher than for the usual spherical collapse model but the Λ CDM model is recovered at high redshifts. Once again major differences take place when $c_{\text{eff}}^2 = 1$. We notice that our results are similar to what was found in Del Popolo et al. (2013b). We can therefore conclude that clustering DE models behave qualitatively as non-clustering DE models in both the spherical and extended spherical collapse model. Shear and rotation terms only oppose to the collapse, without modifying it.

As said before, we have made the assumption that the shear and the rotation terms affect only dark matter. We performed a similar analysis relaxing this assumption and supposing that both dark matter and dark energy are influenced by these additional non-linear terms, then using a single equation for the velocity field, Equation 25. The results obtained are very similar to what presented here, therefore in the following, we will assume that DM and DE have a different peculiar velocity.

4.2 Mass function

Here we study the effect of the shear and rotation terms on the number counts of halos. We assume that the analytical formulation by Sheth & Tormen (Sheth & Tormen 1999; Sheth et al. 2001; Sheth & Tormen 2002) is valid also for clustering dark energy models without any modification (but see also Del Popolo & Gambera 1999; Del Popolo 2006). The mass function critically depends on the linear overdensity parameter δ_c , the growth factor and on the linear power spectrum normalization σ_8 . To properly evaluate the

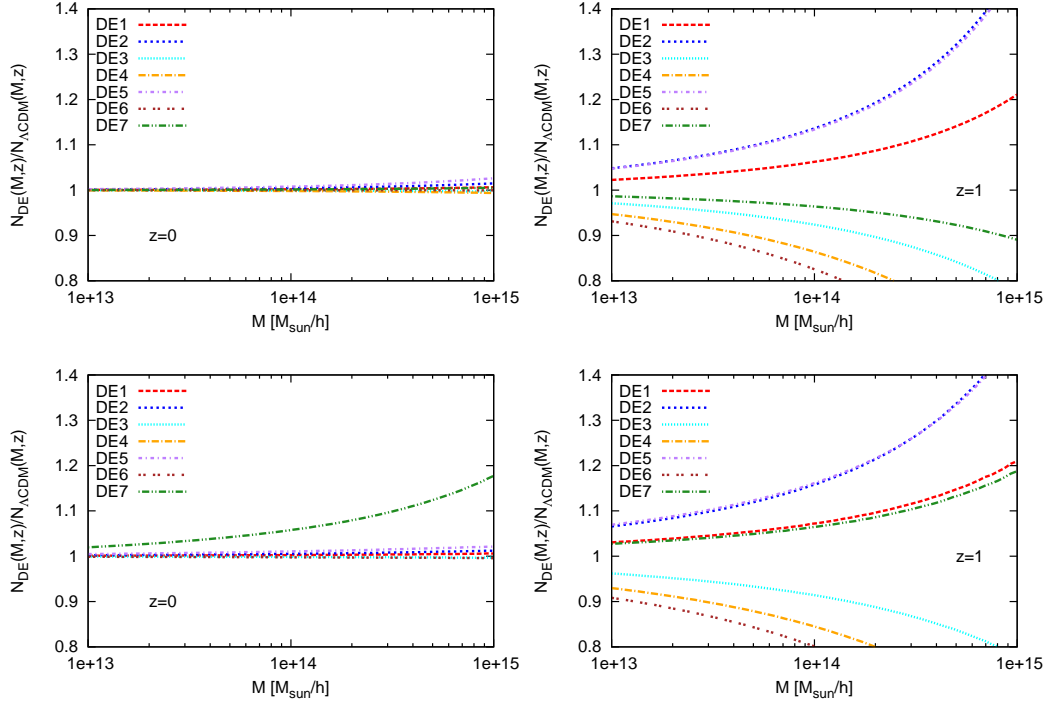


Figure 6. Ratio of the number of objects above a given mass M for halos at $z = 0$ (left panels) and $z = 1$ (right panels) between the DE models and the Λ CDM model. The upper panels show ratios for the usual spherical collapse model while the bottom panels ratios for the extended spherical collapse model. The red dashed curve represents the DE1 model, the blue short-dashed curve the DE2 model, the cyan dotted curve the DE3 model, the yellow dot-dashed curve the DE4 model, the violet dot-short-dashed curve the DE5 model, the brown dot-dotted curve the DE6 model and the green dashed-dot-dotted curve the DE7 model.

effects of the extra terms in the spherical collapse model formalism, we assume that all the models have the same σ_8 at $z = 0$ and to highlight the effect we consider the number of objects at $z = 0$ and $z = 1$ above a given mass M of the halo. We will assume as transfer function for the linear matter power spectrum the functional form given by Bardeen et al. (1986) and $\sigma_8 = 0.776$ as normalization of the power spectrum, in agreement with the most recent measurements (Planck Collaboration et al. 2013b,e).

A comment is necessary at this point to explain the choice of the matter power spectrum normalization. In Batista & Pace (2013) and in Del Popolo et al. (2013b) we had a different normalization for each model, such that all the models would have the same amplitude of perturbations at the CMB epoch ($z \approx 1100$). Here instead we enforce all the models to share the same normalization today. While a model dependant normalization should be in general preferred, here we want to isolate the effect of the shear and rotation terms and analyse their behaviour. If we would also have a different σ_8 for each model, a direct comparison between them would be more difficult.

In Figure 6 we show the ratio of the number of objects above a given mass M between the DE models and the Λ CDM model, restricting the analysis to the case $c_{\text{eff}}^2 = 0$. In the upper (lower) panel we present results without (with) rotation and shear terms. Left (right) panels are for halos at $z = 0$ ($z = 1$). As expected, having the models the same normalization of the matter power spectrum, at $z = 0$ the models have essentially the same number of objects, with very small differences for masses $M \approx 10^{15}$. In particular quintessence models (DE1, DE2, DE5) show a slight excess of structures, while phantom models (DE3, DE4, DE6) show a decrement in the number of structures. The model DE7 is at all

effects identical to the model DE1. Differences grow with redshift and at $z = 1$, they can be few tens of percent, keeping though the same qualitative behaviour. Models with highest differences are those with the equation-of-state parameter differing mostly from $w = -1$. Interestingly enough, the model DE7 shows now a decrement of $\simeq 10\%$ with respect to the Λ CDM model. With the inclusion of the shear and rotation terms, we see a behaviour qualitatively similar to the standard case, with major differences at higher redshifts (due to the time evolution of the dark energy), but with a smaller number of objects, at the level of percent for the phantom models, while quintessence models are largely unaffected. Results are consistent with the time evolution of the linear overdensity factor δ_c (see Figures 2 and 3). It is interesting the case of the model DE7, which shows a very strong dependence on the inclusion of the non-linear terms already at very low redshifts, but its time-dependence is very weak. This is probably due to the change of regime between the phantom and the quintessence one. Note that ratios shown in the second row, are taken with respect to the Λ CDM model evaluated in the ESCM.

Dark energy models with $c_{\text{eff}}^2 = 1$ (see also Del Popolo et al. 2013c) deserve a comment. Due to the major differences between the linear overdensity parameter δ_c of these models with respect to the Λ CDM one, we expect differences already at $z = 0$ and they increase in value at $z = 1$. Qualitatively, the same behaviour is nevertheless recovered, so models DE3, DE4 and DE6 (phantom models) show a decrement in the number of objects. Analogously to the case with $c_{\text{eff}}^2 = 0$, the two additional nonlinear terms just slightly increase differences with respect to the standard case, in agreement with Figures 2 and 3.

As explained before in Sect. 4.1, the linear overdensity pa-

parameter δ_c becomes mass dependant when shear and rotation are included. To evaluate the mass function when these additional non-linear terms are included, we therefore explicitly evaluate the contributions of the shear and rotation terms to δ_c for each mass, so to have an exact evaluation of the mass function. This means that major differences will take place at small masses, while at cluster scales differences between the two different mass functions will be negligible. This is indeed the case in the lower panels of Fig. 6, with the exceptions of the model DE7 as clarified above.

After establishing the effect of the shear and rotation terms on the mass function, we investigate deeper the effects of the clustering of the DE fluid. In this case, the total mass of the halo is affected by dark energy perturbations (Creminelli et al. 2010; Basse et al. 2011; Batista & Pace 2013) and we need to take this into account evaluating the fraction of the halo mass given by the clustering of the dark energy. How and how much dark energy contributes to the halo mass depends on the virialization process, in particular whether dark energy virializes and on which time scale. If the halo mass is modified, then also the merging history (see e.g. Lacey & Cole 1993) must reflect somehow this additional contribution. According to the equation-of-state parameter, DE can add or subtract mass to the total halo mass. An exact treatment of this problem must take into account the nature of the dark energy fluid and its exact virialization process. This is beyond the purpose of this work and we will use an approximate recipe, limiting ourselves to the case in which $c_{\text{eff}}^2 = 0$ and we will assume that DE virializes with DM on the same time scale (see Batista & Pace 2013). In the following we will describe how to evaluate the fraction of DE with respect to the total mass of the halo (Batista & Pace 2013).

As done in Section 4.1, we will assume that $y = R_{\text{vir}}/R_{\text{ta}} = 1/2$ as in the Einstein-de Sitter (EdS) universe. For this model, the virial overdensity Δ_V can be evaluated analytically and in literature two different definitions are usually adopted. The most common one (Wang & Steinhardt 1998), evaluates it at the collapse redshift z_c : $\Delta_V(z_c) = \rho_m(z_v)/\bar{\rho}_m(z_c) \simeq 178$, where z_v is the virialization redshift. According to Lee & Ng (2010) and Meyer et al. (2012), it is more correct to evaluate it at the virialization redshift: $\Delta_V(z_v) = \rho_m(z_v)/\bar{\rho}_m(z_v) \simeq 147$. These values will obviously change in the presence of DE and they depend on the properties of DE (Lahav et al. 1991; Maor & Lahav 2005; Creminelli et al. 2010; Basse et al. 2011).

The fraction ($\epsilon(z)$) of DE mass (M_{DE}) with respect to the DM mass (M_{DM}) is

$$\epsilon(z) = \frac{M_{\text{DE}}}{M_{\text{DM}}} . \quad (26)$$

We define the DM mass as

$$M_{\text{DM}} = 4\pi \int_0^{R_{\text{vir}}} dRR^2 (\bar{\rho}_{\text{DM}} + \delta\rho_{\text{DM}}) , \quad (27)$$

and the DE mass as

$$M_{\text{DEp}} = 4\pi \int_0^{R_{\text{vir}}} dRR^2 \delta\rho_{\text{DE}} (1 + 3c_{\text{eff}}^2) . \quad (28)$$

We label the DE mass as M_{DEp} (see Equation 28) to indicate that in its definition we consider only the contribution coming from the perturbation.

If instead we consider also the background contribution, the mass definition becomes

$$M_{\text{DEt}} = 4\pi \int_0^{R_{\text{vir}}} dRR^2 [(1 + 3w)\bar{\rho}_{\text{DE}} + (1 + 3c_{\text{eff}}^2)\delta\rho_{\text{DE}}] , \quad (29)$$

in analogy with the Poisson equation. In this case there will be also a contribution for the Λ CDM model. However, since the background term varies in time, regardless of the halo formation history, this contribution is not constant and should be interpreted just as a crude estimate of the background DE energy to the halo mass.

In Figure 7 we show the fraction of the DE mass with respect to the DM mass according to the definition used in Equations 28 (upper panel) and 29 (lower panel) for the case $c_{\text{eff}}^2 = 0$ only in the standard spherical collapse model. For a deeper discussion on the mass definition adopted see Batista & Pace (2013). We just show results for the standard spherical collapse model since rotation and shear have a negligible effect on $\epsilon(z)$. In particular for quintessence models the extra terms slightly reduce the DE contribution, while for the phantom models, being $\epsilon(z)$ negative and therefore subtracting mass to the halo, this function is slightly higher, or in absolute values, again slightly smaller. Same result for the barrier crossing model. The effect of the shear and rotation terms is of the order of tenth of percent.

As expected (see Figure 7), quintessence models give a positive contribution to the total mass of the halo while phantom models subtract mass. Differences are of the order of the percent level, except for the model DE5, where differences are up to $\approx 14\%$. In agreement with Batista & Pace (2013), we also notice that the mass correction evaluated with Equation 29 is smaller than when only perturbations are taken into account. Major differences are at $z = 0$ and become null at higher redshifts. This is expected, since $\epsilon(z)$ is significantly different from zero at low redshifts. Exception is once again model DE5. This is due to the fact that its equation of state is very different from $w = -1$. The inclusion of a mass correction term will affect the mass function and major differences will take place for $z \approx 0$, as we show in Figure 8. Differences are again more pronounced for high masses where they can be up to 20% while for low masses it is only of the order of 5% at most. The hierarchy of the models, i.e., how much they are affected, directly reflects the values of the mass correction.

In Figure 8 we show the ratio of the mass function with the new mass definition, $M(1 - \epsilon)$, where ϵ is given by Equation (26). Since we only consider the contribution of the DE perturbations, the cosmological constant does not contribute to the total mass of the system. Quintessence (phantom) models have a lower (higher) number of objects at the low mass end of the mass function and a higher (lower) number of objects at the high mass tail. This can be easily explained taking into account the relative contribution of the DE component to the total mass of the halo (see upper panel of Figure 7). A positive (negative) contribution to the total mass shifts the mass function towards lower (higher) values and, as consequence, we obtain a higher (lower) number of objects. At the high mass end, the contribution of the linear overdensity parameter δ_c dominates, giving the opposite trend with respect to the low mass tail.

We can conclude that shear and rotation terms have in general a negligible contribution also when the mass definition in Equation 28 is adopted.

Similar results apply for the case $c_{\text{eff}} = 1$, where we note that since perturbations in dark energy are negligible, so is the mass correction.

5 CONCLUSIONS

In this work we studied the effect of the inclusion of the term $\sigma^2 - \omega^2$. We analysed its impact in the framework of the spherical collapse model and in particular on the linear overdensity param-

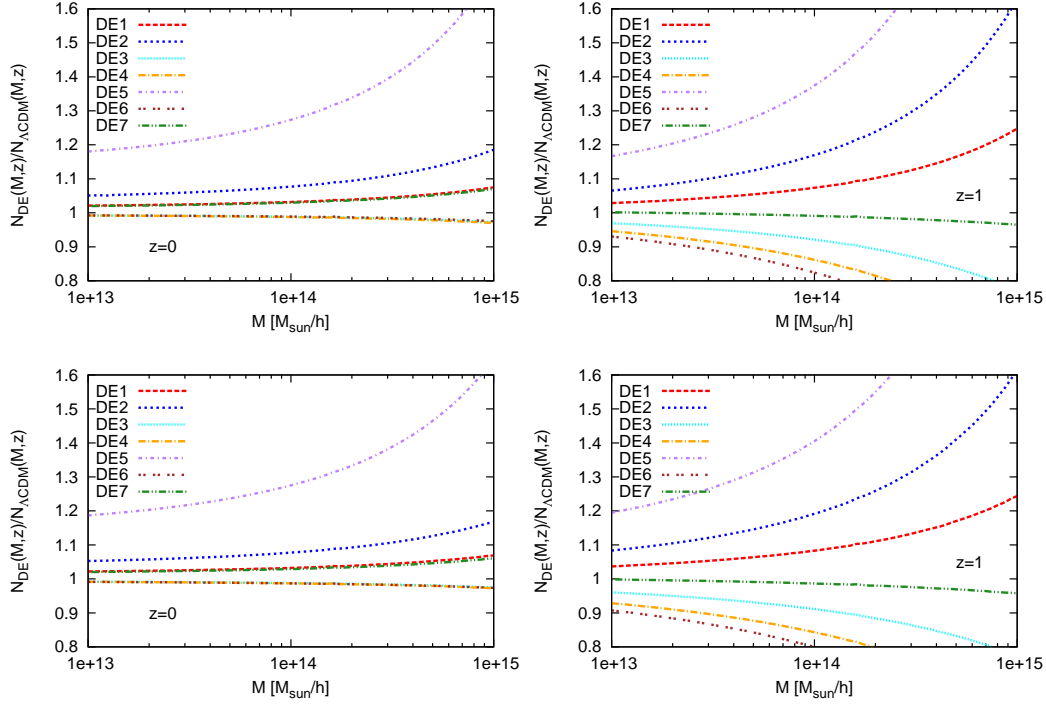


Figure 8. Ratio of the number of objects above a given mass M for halos at $z = 0$ (left panels) and $z = 1$ (right panels) between the DE models and the Λ CDM model, using the mass definition in Equation 28. The upper panels show ratios for the usual spherical collapse model while the bottom panels show the ratios for the extended spherical collapse model. Line styles and colours are as in Figure 6.

ter δ_c and on the virial overdensity Δ_V . The parameter δ_c is one of the ingredients of the mass function and its variation reflects on the mass function and, as consequence, on the number of objects at a given redshift.

We consider dark matter and dark energy component as two fluids described by the respective equation of state and both of them can cluster. In particular, we relate the pressure perturbations to density perturbations for the dark energy component with the effective sound speed parameter c_{eff}^2 , that we assume to be constant and its values were fixed to $c_{\text{eff}}^2 = 0$ and $c_{\text{eff}}^2 = 1$, as currently done in literature.

The $\sigma^2 - \omega^2$ term, being non-linear appears only in the non-linear equation describing the evolution of the peculiar velocity, therefore the growth factor is not affected. We made the assumption that only dark matter is affected by this additional non-linear term, but if we instead suppose that both DM and DE experience shear and rotation, we showed that results are largely unaffected, since for the models studied DE perturbations are subdominant.

We showed that the additional non-linear term opposes the collapse, as for the case in which the dark energy is only at the background level. Opposing the collapse, it makes such that both δ_c and Δ_V have a higher value with respect to the standard spherical collapse model. Increments in the linear overdensity parameter are of the order of 40% for low masses, analogously to what found in Del Popolo et al. (2013b), where the extended spherical collapse model was studied in non-clustering dark energy models. Quintessence models have always a lower value of δ_c , both in the standard and in the extended spherical collapse model. Phantom models instead present higher values, due to the faster expansion of the universe. A similar behaviour is found for the virial overdensity parameter Δ_V .

Differences in the spherical collapse model parameters reflect

obviously in the mass function and in particular in the number of objects above a given mass (see Section 4.2). To properly evaluate the effect of the additional term, we use the same normalization of the linear matter power spectrum for all the models. Moreover, considering the number of objects above a given mass, does not introduce any geometrical dependence on the results that will therefore depend only on the particular model considered (DE equation-of-state parameter and effective sound speed). Comparing results in the ESCM with the standard SCM, we notice that the differences are in general small, of the order of the percent for all the models considered in this work.

When dark energy clusters, following Batista & Pace (2013), we speculate that the halo mass can be modified by the inclusion of the dark energy perturbation into its definition. We therefore evaluate the correction to the halo mass and we found that this is generally of the order of few percent at low mass (but higher on cluster scales) and its sign (being positive or negative) depends on the equation of state of the dark energy component. The shear and rotation terms slightly modify this function, making it closer to zero when these terms are taken into account. Due to the small value of this correction factor, modifications in the number of objects is also small.

We can therefore conclude that effects of rotations in clustering dark energy models are modest and comparable to what found in Del Popolo et al. (2013a,b) for non-clustering dark energy models. Hence we may also expect that the effect of clustering dark energy in more realistic models of structure formation can be well described by the usual spherical collapse model.

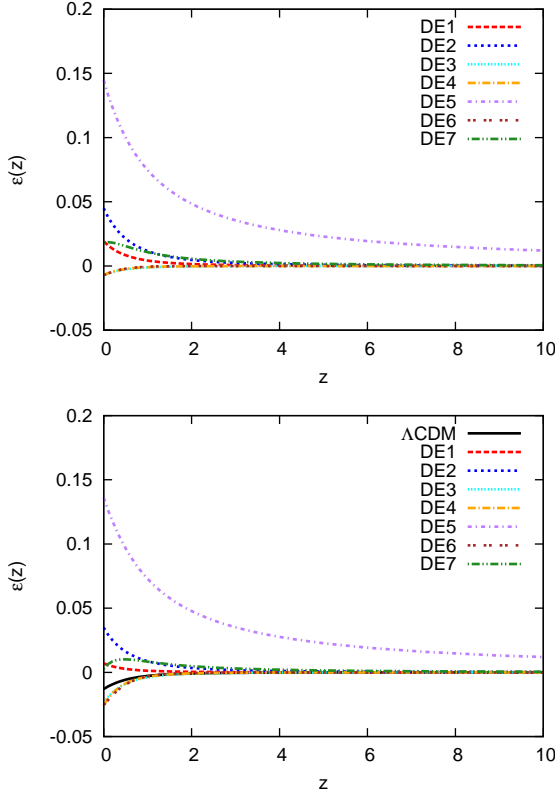


Figure 7. Fraction of the DE mass with respect to the DM mass according to the definition of Equations 28 (upper panel) and 29 (lower panel). The black solid curve shows the Λ CDM model, the red dashed curve the DE1 model, the blue short-dashed curve the DE2 model, the cyan dotted curve the DE3 model, the yellow dot-dashed curve the DE4 model, the violet dot-short-dashed curve the DE5 model, the brown dot-dotted curve the DE6 model and the green dashed-dot-dotted curve the DE7 model.

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